

# Anomaly Detection in Video Surveillance: A Novel Approach Based on Sub-Trajectory

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**Abstract**— In video surveillance based on motion trajectory, a moving object is considered abnormal if the distance from it to trajectory is greater than a threshold. If a trajectory is abnormal with sub-trajectory, it will be abnormal with entire trajectory. In this paper, a novel approach proposes to detect abnormal based on sub-trajectory with modified Hausdorff distance. Moreover, the anomaly detection based on sub-trajectory that can be done with complete trajectory and incomplete trajectory. The proposed technique is evaluated with 1000 datasets and each dataset consists of 260 trajectories. The result show that the technique detect abnormal with the time faster.

**Index Terms**—anomaly detection, sub-trjectory, Hausdorff distance, motion trajectory.

## I. INTRODUCTION

Continuous monitoring mission and ensure credibility with a large number of video streams is a challenge for the operating of monitoring system. Automatic video surveillance can help reduce the cost of labor, as well as giving the appropriate notice as necessary. Because of these, anomaly detection in video surveillance has attracted many researchers in the field of computer vision.

There are many methods of anomaly detection, but it can be classified into two groups, based on the characteristics of the line video images [1] [2] and based on analysis of the motion trajectory of object [3] [4]. In recent years, the method of analysis based on the motion trajectory has received a lot of attention of the researchers [5] [6] [7].

The anomaly detection technique based on trajectory analysis was done by clustering the trajectory to eliminate outliers [8]. Then the abnormal is detected by calculating the distance of the new trajectory to the center of clusters. The processing model is shown in Figure 1.

Most of the proposed algorithm detect abnormal with complete trajectory. This is clearly a disadvantage in automatic monitoring applications with real-time.

In this paper, we propose a novel technique with modified Hausdorff distance [9, 10] to satisfy the properties of a metric, and segment a trajectory into sub-trajectories based on the changing of the velocity [11, 12, 13, 14]. Anomaly detection algorithm in video surveillance based on motion trajectory is proposed in the paper by modifying Hausdorff distance to detect

abnormal in sub-trajectories. The algorithm can detect abnormal with incomplete trajectory with the aim to reduce the detection time, so the technique can meet the video surveillance system in real-time.

The rest of this paper is organized as follows: Section II presents system design. Section III evaluates the result of the method. Finally, Section IV concludes the paper and figures out the future works.

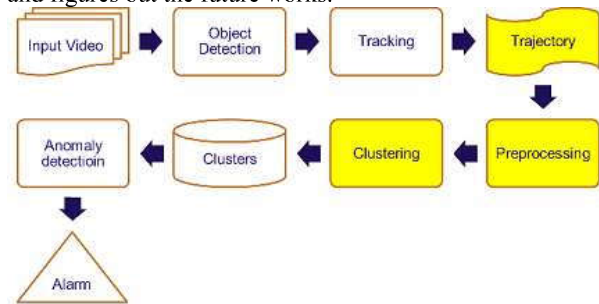


Figure 1. The processing model for anomaly detection

## II. SYSTEM DESIGN

### A. Some Definitions

#### 1) Definition 1: Distance from a point to a set

Let  $(X, d)$  be complete metric space and let  $H(x)$  be compact subset of  $X$ . With  $x \in X$  and  $B \in H(X)$ , The distance from a point to a set is defined as follows:

$$d(x, B) = \min\{d(x, y) : y \in B\}$$

#### 2) Definition 2: Distance between two sets

Let  $(X, d)$  be complete metric space. With  $A, B \in H(X)$ , The distance from set  $A$  to set  $B$  is defined as follows:

$$d(A, B) = \max\{d(x, B), x \in A\}$$

#### 3) Definition 3: Hausdorff Distance

Let  $(X, d)$  be complete metric space. The Hausdorff distance from set  $A$  to set  $B$  is defined as follows:

$$h(A, B) = \max\{d(A, B), d(B, A)\}$$

#### 4) Theorem 4

$h$  is metric on  $H(x)$ .

#### Proof.

If  $h$  satisfies reflexivity, symmetry and triangle inequality, then  $h$  is metric on  $H(x)$ .

#### (i) Reflexivity.

$$h(A, A) = \max\{d(A, A), d(A, A)\} = \max\{d(a, A) : a \in A\} = 0.$$

(ii) Symmetry.

$$h(A, B) = \max\{d(A, B), d(B, A)\} = \max\{d(B, A), d(A, B)\} = h(B, A)$$

(iii) Triangle inequality.

$$A \neq B \neq C \in H(x) \Rightarrow \text{any } a \in A, a \notin B : d(a, B) > 0 \Rightarrow h(A, B) \geq d(a, B) > 0$$

$$\forall a \in A \text{ and } \forall c \in C, \text{ we have } d(a, B) = \min\{d(a, b) : b \in B\} \leq \min\{d(a, c) + d(c, b) : b \in B\}$$

$$\Rightarrow d(a, B) \leq d(a, C) + \min\{d(c, b) : b \in B\}, \forall c \in C$$

$$\Rightarrow d(a, B) \leq d(a, C) + \max\{\min\{d(c, b) : b \in B\} : c \in C\}$$

$$\Rightarrow d(a, B) \leq d(a, C) + d(C, B)$$

$$\text{Thus, } d(A, B) = \max\{d(a, B) : a \in A\} \leq d(a, C) + d(C, B) \leq d(A, C) + d(C, B)$$

$$\text{Similarly, we have } d(B, A) \leq d(B, C) + d(C, A)$$

$$h(A, B) = \max\{d(A, B), d(B, A)\}$$

$$\leq \max\{d(A, C) + d(C, B), d(B, C) + d(C, A)\}$$

$$\leq \max\{d(A, C), d(C, A)\} + \max\{d(C, B), d(B, C)\}$$

$$\leq h(A, C) + h(C, B)$$

### 5) Definiton 5: Motion Trajectory

Let  $O = \{t_1, t_2, \dots, t_n\}$  be the motion trajectory of object O. Sequence  $\langle t_1, t_2, \dots, t_n \rangle$  presents the position of object O at the time  $t_1, t_2, \dots, t_n$ .

Figure 2 shows that the motion trajectory of the object was obtained in the process of tracking objects.



Figure 2 - The motion trajectory of the object

### 6) Definiton 6: Hausdorff Distance Between Two Trajectories.

Given two trajectories  $A = \{a_1, a_2, \dots, a_n\}$  and  $B = \{b_1, b_2, \dots, b_m\}$ .

Distance between two trajectories  $h(A, B)$  is defined by (1)

$$h(A, B) = \max\{d(A, B), d(B, A)\} \quad (1)$$

Wherein,  $d(A, B)$  and  $d(B, A)$  are calculated by (2)

and (3).

$$d(A, B) = \max\{d(a_i, B), a_i \in A\} \quad (2)$$

$$d(B, A) = \max\{d(b_i, A), b_i \in B\} \quad (3)$$

And  $d(a_i, B)$  and  $d(b_i, A)$  are calculated by (4) and (5).

$$d(a_i, B) = \max\{d(a_i, b_j), b_j \in B\} \quad (4)$$

$$d(b_i, A) = \max\{d(b_i, a_j), a_j \in A\} \quad (5)$$

Distance  $d(a_i, b_j)$  is calculated by (6).

$$d(a_i, b_j) = d_e(a_i, b_j) + \gamma d_0(a_i, b_j) \quad (6)$$

Wherein,  $\gamma$  is the parameter to adjust the weight of the moving direction,  $d_e(a_i, b_j)$  and  $d_0(a_i, b_j)$  are calculated by (7) and (8).

$$d_e(a_i, b_j) = \sqrt{(x_i^a - x_j^b)^2 + (y_i^a - y_j^b)^2} \quad (7)$$

$$d_0(a_i, b_j) = 1 - \frac{v_{a_i} \cdot v_{b_j}}{|v_{a_i}| \cdot |v_{b_j}|} \quad (8)$$

Wherein, the velocity  $v_{a_i}$  and  $v_{b_j}$  are calculated by (9) and (10).

$$v_{a_i} = (x_i^a - x_{i-1}^a, y_i^a - y_{i-1}^a) \quad (9)$$

$$v_{b_j} = (x_j^b - x_{j-1}^b, y_j^b - y_{j-1}^b) \quad (10)$$

### 7) Definiton 6: Route

Given a collection of trajectories  $R_i = \{O_1, O_2, \dots, O_r\}$  and threshold  $\sigma$ .  $R_i$  is called a route if  $h(O_i, O_j) \leq \sigma, \forall O_i, O_j \in R_i$

The tracking of motion established the route is shown in Figure 3.



Figure 3. The tracking of motion objects

### 8) Definiton 8: Anomaly Definition

When a regional surveillance by camera, objects moving in the trajectory often made certain routes. An object is called abnormal movement if it does not belong to any given trajectory groups.

Given the trajectory  $T^* = \{t_1, t_2, \dots, t_n\}$  and the routes  $R = \{R_1, R_2, \dots, R_k\}$ . The distance from  $T^*$  to  $R$  is calculated by (11).

$$d_{\text{detect}}(T^*, R) = \min_{i=1, \dots, k} \{h(T^*, R_i)\} \quad (11)$$

With  $d_{\text{max}}$  is a given threshold, if  $d_{\text{detect}}(T^*, R) > d_{\text{max}}$ , then  $T^*$  is a abnormal trajectory.

### 9) Definiton 9: Sub-trajectory

Route segmentation is based on the velocity of moving objects. Each route segmentation is called a sub-trajectory. The segment points are specified when the velocity is greater than the threshold. The velocity of object is calculated by (12).

$$v_i^r = \min\left(\frac{v_i^x - v_{i-1}^x}{v_{i-1}^x}, \frac{v_i^y - v_{i-1}^y}{v_{i-1}^y}\right) \quad (12)$$

Wherein,  $v_i^x$  and  $v_i^y$  are the velocity along the x-axis and the y-axis, respectively.  $v_i^x$  and  $v_i^y$  are calculated by (13) and (14).

$$v_i^x = x_i - x_{i-1} \quad (13)$$

$$v_i^y = y_i - y_{i-1} \quad (14)$$

Let  $\text{Seg} = \{\text{seg}_1, \text{seg}_2, \dots, \text{seg}_u\}$  be the segment points of the trajectory O ( $1 < \text{seg}_i < n, 1 < u < n$ ). Trajectory O is divided into  $u+1$  segments and is shown as follows:

$$O = \{t_1, t_2, \dots, t_{\text{seg}_1}, t_{\text{seg}_1+1}, \dots, t_{\text{seg}_2}, \dots, t_{\text{seg}_u}, t_{\text{seg}_u+1}, \dots, t_n\}$$

The trajectories  $SO_i = \{t_1, t_2, \dots, t_{seg_i}\}$  are called sub-trajectories. Figure 4 shows the trajectory is divided into the sub-trajectories.



Figure 4 – Segment points

10) *Theorem 10*

Let P be the medium route of a route.

$$P = \{p_1, p_2, \dots, p_{seg_1}, p_{seg_1+1}, \dots, p_{seg_2}, \dots, p_{seg_u}, p_{seg_u+1}, \dots, p_n\}$$

Wherein,  $seg = \{seg_1, seg_2, \dots, seg_u\}$  are the segment points of P ( $1 < u < n$ ).  $SP_i = \{t_1, t_2, \dots, t_{seg_i}\}$  are the sub-trajectories of P.

If the trajectory T\* is abnormal with the segment i ( $1 \leq i \leq u$ ), then the trajectory T\* will be abnormal with all segments l ( $1 < l \leq u$ ).

**Proof.**

T\* is abnormal with the segment i, so  $d_{detect}(T^*, SP_i) = h(T^*, SP_i) > d_{max}$

Suppose T\* is not abnormal with the segment l ( $1 < l \leq u$ ), we have  $d_{detect}(T^*, SP_l) < d_{max}$ . Moreover,  $d_{detect}(T^*, SP_l) = \min\{h(T^*, SP_l), h(T^*, SP_i)\} < d_{max}$ , hence  $h(T^*, SP_l) < h(T^*, SP_i) < d_{max}$ . This is contrary to the hypothesis. Hence, theorem completes the proof.

B. *Anomaly Detection Based on Sub-Trajectory*

In this paper, we propose a novel approach to detect anomaly based on sub-trajectory. Our method includes two phases.

1) *The first phase*

Let us denote as follows:

- $R = \{R_1, R_2, \dots, R_k\}$  is the routes
- $r_i$  is the number of the route  $R_i$  ( $1 \leq i$ )
- $O_j^i$  is the trajectory of the route  $R_i$  ( $1 \leq i \leq k, 1 \leq j \leq r_i$ )
- $P = \{P_1, P_2, \dots, P_k\}$  is the medium routes of the routes.
- $P_i$  is the medium route of the route  $R_i$ .
- $SO_j^i$  is the  $j^{\text{th}}$  sub-trajectory of the medium route  $P_i$

$$SO_j^i = \{P_i(P_1, P_2, \dots, P_{seg_j})\}$$

This phase is carried out as follows:

- Step 1: Create the trajectory group of the routes
- Step 2: Calculating the medium routes by (15)

$$P_i = \left\{ \frac{1}{n} \sum_{j=1}^n O_j^i(t_j) \right\} \quad (15)$$

-Step 3: Specify sub-trajectories based on the medium route.

- Step 4: Calculating the threshold  $d_{max}$  by (16)

$$d_{max} = \min_{i=1..k} \max_{j=1..r_i} h(O_j^i, P_j^i) \quad (16)$$

2) *The second phase*

Based on theorem 10, we propose the algorithm to detect anomaly based on sub-trajectory as follows:

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Algorithm: Anomaly Detection Based on Sub-Trajectory of Route (ADB-STR)

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**Input:**

- $U_{max}$ : The maximum number of sub-trajectories
- k: The number of routes
- $d_{max}$ : The value of threshold
- $\{SO_j^i\} (i=1..k); (j=1..u_{max})$ : set of sub-trajectories
- T\* is the check trajectory

**Output:**

- True if detect abnormal
  - False if not detect abnormal
- 

```

j=1;
Abnormal=False;
While (j ≤ umax and Abnormal =False)
{
    d = min(h(T*, SO_j^i));
    if (d > dmax) then Abnormal = True;
    j++;
}
return Abnormal;

```

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III. EVALUATION

For experiment, we have collected 1,000 datasets from [15]. Each dataset consists of 260 trajectories which includes 250 normal trajectories and 10 abnormal trajectories. We divided 260 trajectories into two sets, the first set called training set contains 200 normal trajectories, the second set called testing set contains 60 trajectories which includes 50 normal trajectories and 10 abnormal trajectories.

Experimental procedure was divided into two phases via Matlab R2013a. The first phase was divided into 4 steps aim to specify the threshold  $d_{max}$  with training set. The second phase will detect abnormal with testing set.

A. *The First Phase*

- Step 1: From training set (200 normal trajectories), we divided into 5 groups of trajectory as shown in Figure 5.

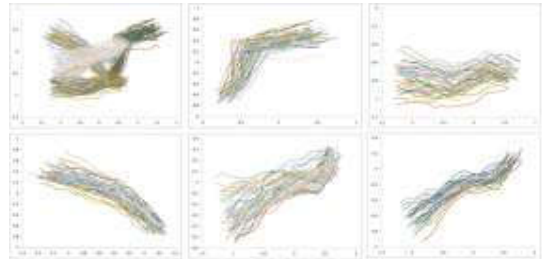


Figure 5 – 5 groups of trajectories

- Step 2: The medium routes are calculated by equation (15) as shown in Figure 6.

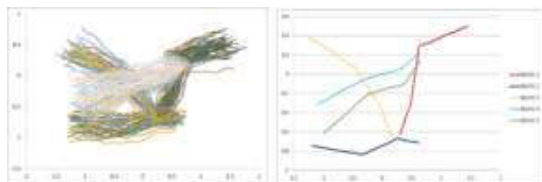


Figure 6 – The medium routes

- Step 3: Segmenting the medium routes to determine sub-trajectories based on the changing of velocity. The result is shown in table I and Figure 7.

TABLE I. THE RESULT OF THE ROUTE SEGMENT

Groups	Segments	The segment point
Route 1	2	10
Route 2	2	5
Route 3	2	10
Route 4	2	5
Route 5	2	5

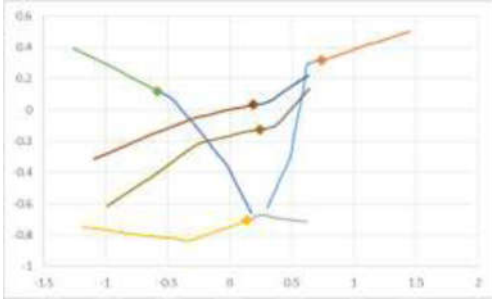


Figure 7 – sub-trajectories

- Step 4: Determining the threshold  $d_{max}$  by (14).

#### B. The Second Phase

Detecting anomaly trajectory by algorithm ADB-STR with testing set. The result of anomaly trajectory is shown in Figure7, the abnormal trajectories is drew by red. The results of testing are fully appropriate to the experimental results of Piciarelli [15] and Laxhammar [16]. However, the detection time is faster than the detection time of Piciarelli [15] and Laxhammar [16], because the proposed method mustn't process all medium routes. From the result of table I, we show that the abnormal trajectory is detected at the 10th segment point, in the worst case.

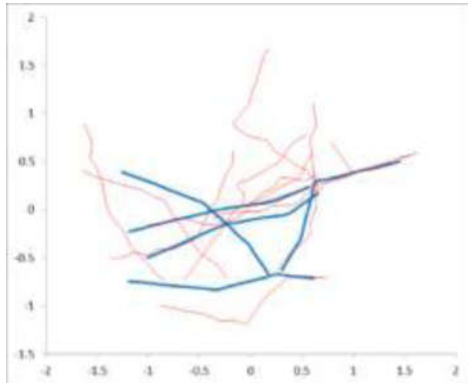


Figure 7 – Abnormal trajectories

#### IV. CONCLUSIONS

In this paper, we have proposed a new technique to detect anomaly in video surveillance effectively. In the proposed technique, the system model is built to detect anomaly based on sub-trajectory by segmenting the medium routes and modified Hausdorff distance. The result of the proposed technique show that the detection time is very fast, so the technique can apply for the real-time video surveillance systems.

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